PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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On the Passage of Heat between Metal Surfaces and Liquids in Contact with Them

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IV. *On the Passage of Heat between Metal Surfaces and Liquids in contact with them.*

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Communicated by Professor OSBORNE REYNOLDS, *F.R.S.*

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Introduction.

THE determination of the rate of transmission of heat from the surface of a heated metal to water in contact with it, or from hot water to a colder surface, is a problem of some difficulty, but is of such great importance in the theory of boilers and surface condensers, that further investigation would seem to be justified, especially as, although the theory of the subject has been much studied, yet owing to practical difficulties, the constants involved, so far as the author is aware, have not been definitely determined for cases which occur in practice.*

PECLET'S experiments on the rate of transmission of heat from *water to water* across a metal plate,† throw much light on the question. His experiments were made to determine the thermal conductivities of various metals by measuring the heat passed through a metal plate, one side of which was exposed to steam and the other to water which was kept agitated by a stirrer. In these experiments, PECLET found that the heat transmitted was sensibly independent of the nature and thickness of the metal used, the conclusion being that on each side of the plate there was a film of water through which the heat was transmitted by conduction, and that compared to these, the thermal resistance of the plate was small.

This difficulty was overcome by an arrangement consisting of revolving brushes in contact with each side of the plate, so as to prevent the formation of a film on the surfaces, and by keeping the water in a violent state of agitation. In this way it was possible to keep the surfaces of the plate at the same temperature as the water in contact with them, and the conductivities of metals determined by this method agree with carefully determined conductivities obtained by other methods.

PECLET further pointed out, that before the brushes were used to prevent the

* RANKINE'S 'Steam Engine,' p. 266.

† 'Traité de la Chaleur,' p. 388.

formation of a film, the rapidity of the agitation of the water by the stirrer had a marked effect on the amount of heat transmitted. This fact has been often observed in experiments on steam boilers, in which the rate of evaporation has been shown to depend upon the rapidity of the convection of the water in the boiler.

Hence, in order to determine the rate of transmission for the case of the ordinary heating surfaces of boilers, it would be necessary to first determine the rate of convection of the water to and from the surface. The difficulty of measuring the convection in such a case is very great, and seemed an effectual bar to all experiment.

A means, however, of measuring the rate of convection appeared in the case of water flowing through metal pipes, at fairly high velocities, which could be determined, and it is to this case that the experiments described are confined.

The theory of the transfer of heat under these conditions has been stated by Professor OSBORNE REYNOLDS.* According to this theory, the heat carried off by any fluid from a surface is proportional to the internal diffusion of the fluid at or near the surface, that is, for a given difference of temperature between the fluid and the surface.

Professor REYNOLDS further points out that the rate of this diffusion will depend on—

(1.) The natural internal diffusion of the fluid when at rest.

(2.) The eddies caused by visible motion which mix the fluid up and continually bring fresh particles into contact with the surface, and that the combined effect of these two causes may be expressed as follows :—

$$H = At + B\rho vt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where t is the difference of temperature between the surface and the fluid, ρ is the density of the fluid, v its velocity, and A and B constants depending on the nature of the fluid; H being the heat transmitted per unit of surface in unit time. In the same paper experiments were described giving evidence in favour of the truth of the above theory.

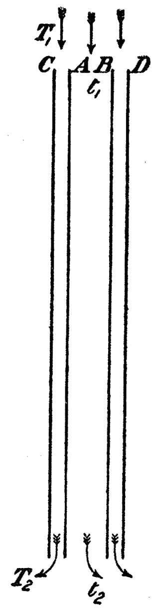
The chief difficulty in any experimental determination of the rate of transmission in metal pipes, lies in the fact that the temperature of the surface of the pipe varies from point to point along the pipe, and again tends to adjust itself by lateral conduction along the pipe. Hence, in order that any definite results may be obtained, it is necessary that the temperature of the pipe shall be constant throughout its length. It occurred to the author that this result might be obtained in the following way :—

In fig. 1, AB represents a tube placed vertically, and surrounded by a second tube CD, the annular space between them being used as a water-jacket. Now, if hot water at a temperature T_1 initially, flow down the water-jacket, and cold water, at a temperature t_1 flow down the pipe, then heat will be transmitted through the

* 'Proceedings, Manchester Lit. and Phil. Soc.,' 1874, p. 9.

walls of the pipe to the cold water, and if the *quantities* of water be the same in each case, the fall of temperature of the jacket water will be equal to the rise of temperature of the water flowing through the pipe, if means are taken to prevent the escape of heat from the jacket water to the outer walls.

Fig. 1.



In this way, although the range of temperature from water to water is diminishing, yet the *mean* value of $(T + t)$ is the same at all cross sections.

Now the temperature of the wall of the pipe at any cross section will not necessarily be a mean between the values of T and t at that section, but if the total fall of temperature from one end of the pipe to the other is small, say not more than 6° C., then *under certain conditions of flow which will be stated*, we may fairly assume that the ratio of the differences of temperature between (jacket water and wall) and between (wall and water flowing through pipe) is constant for the whole length of the pipe, and hence that the temperature of the pipe is constant throughout its length.

As regards the conditions of flow it is necessary to point out that if the motion in the pipe or the jacket is "steady," *i.e.*, the water flows in stream-lines parallel to the axis of the pipe, then the temperature of the water cannot be considered as uniform across any section of the pipe, and might vary considerably.

In order to avoid this condition of flow, all the experiments were made at velocities considerably higher than the critical velocity of water for the pipe in question ; this "critical" velocity, as determined by Professor REYNOLDS' experiments,* being given by the expression

$$V_c = \frac{1}{278}\frac{P}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

where

D = diameter of pipe in metres,
T = temperature of the water,
$P = (1 + \cdot 0336\,T + \cdot 000221\,T^2)^{-1}$,
V_c = critical velocity in metres per second.

Equation (2) gives the critical velocity for the smooth lead pipes used in those experiments, and it may be assumed that the critical value of the velocity for smooth copper pipes does not vary greatly from this.

Under these conditions, and using an apparatus as described above, it seemed possible to study experimentally the transmission of heat from metal to water, and water to metal, at varying velocities and ranges of temperature, by careful observations of the initial and final temperatures of the water and the temperatures of the surface.

* 'Phil. Trans.,' 1883, p. 976.

The apparatus being made, before more complicated experiments were tried, it was necessary to test the truth of the above assumption that, at velocities greater than the critical value for the pipe, the temperature at any cross section was constant at all points in the section, not immediately adjacent to the surface of the hot pipe.

To test this, the pipe and jacket were fixed vertically, with hot water at about 70° C. flowing up the jacket, and water at 20° initially flowing up the pipe. A cap was fitted to the end of the pipe, having a small pipe, of half the diameter of the outer one, projecting about 1 centim. into it, as shown in fig. 2.

In this way the water at the section AB was divided, the inner portion passing into the chamber C, and the outer portion into the chamber D, in which the temperatures were taken by thermometers.

It was found impossible to detect any difference in temperature between the water taken from the centre of the pipe and that from the outside, and this even in cases when the velocity was considerably below the critical value, thus showing that when the temperature of water flowing through a pipe is continually changing the motion is unsteady, even although the velocity is *below* the critical value.

Fig. 2.

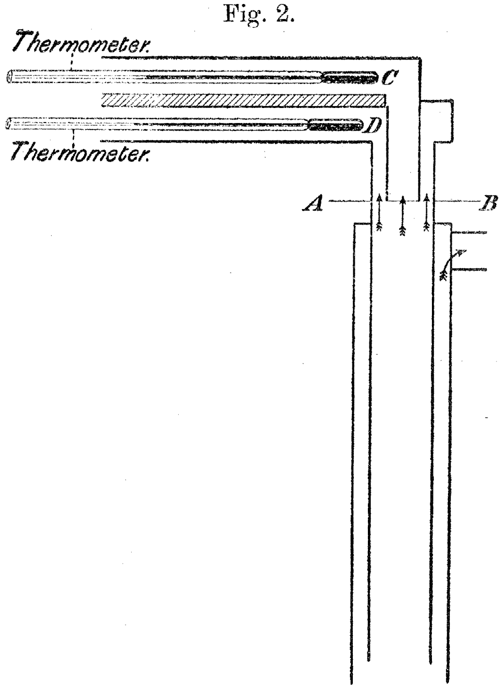
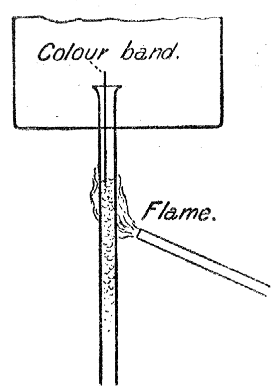


Fig. 3.



This may be illustrated by the following experiment. A glass jar, filled with water, and having a glass pipe fixed to it, as shown in fig. 3, and through which water may be drained, is allowed to stand until the eddies in the water have died out.

A tap is then opened at the bottom, and the water allowed to flow down the pipe.

If now a streak of highly-coloured water be allowed to pass into the pipe from the tank, then, if the velocity is sufficiently low, this extends in a straight line down the tube. (This method of showing the stream-lines was used by Professor REYNOLDS in his experiments,* and has been fully described.) If now the flame of a Bunsen be applied to the outside of the glass pipe, it will be noticed that the streak of colour soon begins to flicker, and finally breaks up into eddies, and this for a very small rise of temperature of the water.

This experiment seems to explain the equality of temperatures observed in the case of the water taken from the heated copper tubes. It was found, however, that experiments made at velocities below the critical value were unreliable, owing to the rapid alternations between steady and unsteady motion which went on in the pipe. This was shown in the following way. When the velocity was above the critical value, then for the given temperature of the surface, the final temperature of the water was perfectly steady. If now the velocity was reduced until near its critical value, it was observed that the mercury in the thermometer stem suddenly began to oscillate rapidly through a range of one, or sometimes two degrees, and that the time of these oscillations was about two seconds.

The table shows at what velocity the unsteadiness comes in.

Diameter of pipe.	Temperature of surface.	Initial Temperature of water.	Final Temperature of water.	Velocity. In centims. per second.
1·39	39·6	18·00	22·6	69·0
1·39	39·6	18·00	22·75	58·1
1·39	39·6	18·00	22·91	43·6
1·39	39·6	18·00	23·2—23·15	28·6
			(Unsteadiness	beginning.)
1·39	39·6	18·00	23·4—23·8	18·00
			(Mercury oscil	lating rapidly.)

Now assuming that the critical velocity for the pipe used is given by

$$V_c = \frac{P}{278D} \text{ metres per second,}$$

then for a temperature of 23°

$$V_c = 0·137,$$

so that the unsteadiness observed from the thermometer readings becomes marked at about a velocity of

$$1·3\ V_c \text{ to } 1·4\ V_c.$$

* 'Phil. Trans.,' 1883, p. 942.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

This result agrees remarkably with the results of Professor REYNOLDS' experiments on the change of the law of resistance in pipes, in which he found a range of unsteadiness in the *pressure* lying between

$$V_c \text{ and } 1{\cdot}3\ V_c.$$

Description of the Apparatus.

For the purpose of the experiments, three drawn copper tubes were obtained, the thickness of the metal being ·08 centim., the length 48 centims., and the internal diameters being 1·39, 1·07, and ·736 centims. respectively.

As it was necessary that the velocity of the jacket water should be as high as possible, in order that the motion in the jacket might be "eddying" (the critical velocity for such a case not having been yet determined), the distances between the outer surfaces of the pipe and the inner surface of the jacket was made as small as possible, consistent with the possibility of getting the required amount of water to flow through under the available pressure.

The jacket pipes were of brass, the width of the jacket space being

·165, ·065, ·16 centim.

for the three cases.

To insure a uniform density of the water at any cross section, it was necessary to place the pipes in a vertical position, and, to make the motion of the water as unstable as possible, the water flowed downwards in each case.

The water used was obtained from a large tank in the tower of the College buildings, the head available being about 100 feet, which remained practically constant throughout the experiments. It was found that the supply from the Town's main was useless, owing to the varying pressure in the mains causing the flow to be unsteady.

Measurement of the Water.

To estimate the quantities of water passing through the pipe and jacket, two meters were required which would give correct values of the amount of water passing through them at any instant, and which should be sensitive, *i.e.*, would indicate *at once*, the change in the flow due to an adjustment of the water valves.

In the early experiments, the water ran into cylindrical vessels with open tops, and thin lipped orifices, through which the water was discharged; the discharge being estimated from the head of water in the vessel.

The objection to this form of meter is that the change in the "head," due to an alteration of the valve, takes place slowly, in some cases nearly a minute elapsing before the head of water in the vessel attains its new position. Consequently the adjustment of the valves to obtain exactly equal amounts of water through the pipe and jacket became very difficult and tedious.

To remedy this defect, two meters were made according to suggestions made by Mr. Foster of Owens College, and these proved very successful. The form of the meter is shown in fig. 5, and consists of a cylindrical tin box, 14 centims. diameter by 7 centims. deep. The water enters at the centre A, and flows radially over the flat plate BC, then radially inwards, guided by the radial vanes to the central orifice, which is re-entrant. When in use the meter is always full, and the motion is very steady.

The head is measured by the column of water in the small glass tube EF, and the flow read off on the scale GH, which is calibrated at intervals by experiment. It was found that on any re-adjustment of the water valves, the level of the water in EF almost instantly took up its proper position for the altered flow, so that it became an easy matter to set the two valves to give an equal flow of the desired amount.

The circular orifices were made of the thinnest sheet brass procurable, soldered to brass plugs, which could be screwed into the bottom plate of the meter. A set of three pairs of orifices was made, and the scale calibrated for each by experiment.

Method of Heating the Water.

As it was necessary to be able to adjust the initial temperature of the jacket water and pipe water to any required value, two copper coils were made, out of $\frac{3}{8}''$ tubing, the length used being about 8 feet for each coil. These were contained in cast-iron cylinders, which were connected to a steam boiler, and were provided with suitable cocks and drains.

During every experiment it was found necessary to maintain the pressure in the boiler constant, as a variation of two pounds on the square inch in the boiler pressure had a considerable effect on the final temperature of the water passing through the coils.

By using steam in the cylinders surrounding the coils at 60 lbs. per square inch pressure, it was possible to raise as much as 25 lbs. of water per minute through a range of 50° C.

The arrangement of the heating coils is shown in fig. 4.

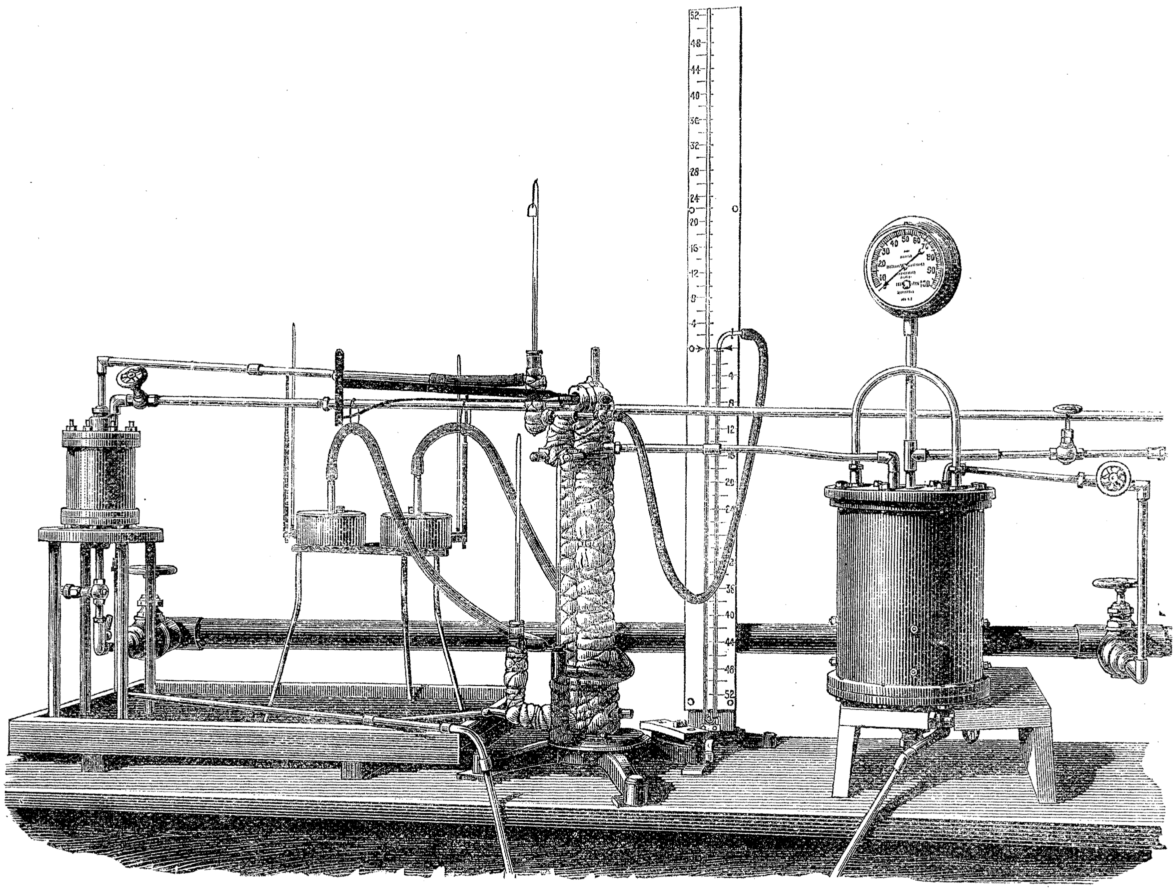
Measurement of the Surface Temperature of the Pipes.

In the preliminary experiments it was attempted to measure the temperature of the surface by means of a thermo-electric couple. As the wires had to be taken through the hot water in the jackets, considerable difficulty was experienced in insulating them, and although several attempts were made, consistent results could not be obtained, so that the method was abandoned.

A satisfactory means of measuring the surface temperature was found in the following way.

Since the thickness of the metal was only 0·08 centim., and the heat transmitted per square centim. per second was not in any case greater than 10 thermal units, and in the majority of the experiments was less than 5 thermal units, then, from the known conductivity of copper, the fall of temperature from one side of the wall to the

Fig. 4.

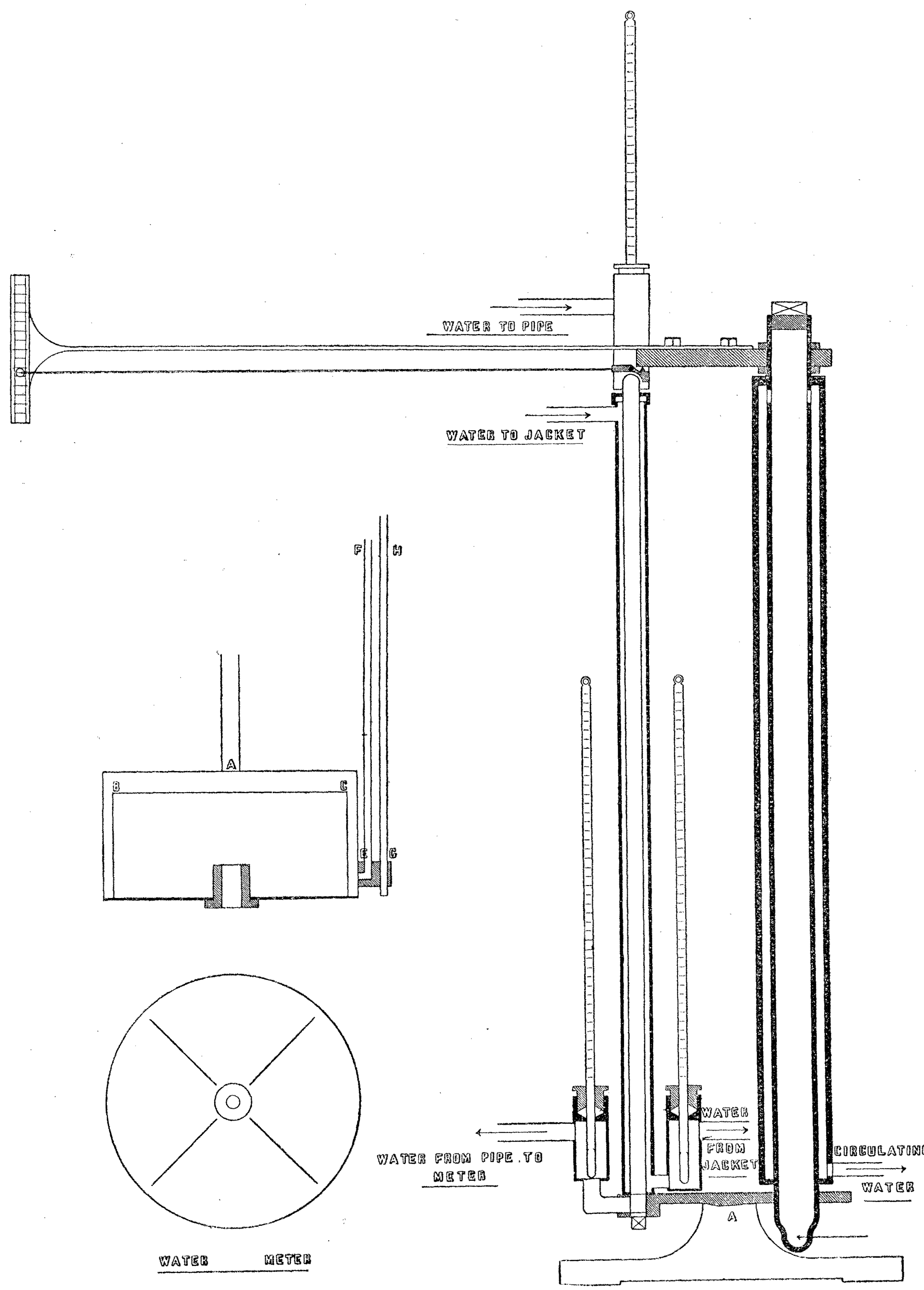


other would in most cases be less than 0·5° and would never exceed 1°. Thus, if the mean temperature of the pipe be determined, then by applying a small correction to this, depending on the amount of heat transmitted, the surface temperature can be found.

The mean temperature of the pipe was found by observing the actual elongation of the copper pipe by means of an extensometer, the arrangement being shown in fig. 5.

A is a cast-iron standard, the upper face of which is planed, and into which the copper tube is screwed, the connexion being as thin as possible, in order to prevent loss of heat from the pipe to the metal of the standard. At a distance of $3\frac{1}{2}$ inches from the copper pipe a wrought-iron pipe is screwed into the casting, which is also surrounded by a water-jacket as shown.

Fig. 5.



During a set of experiments, water at a constant temperature is circulated up through the inner pipe passing into the jacket through the holes shown at the top, and then down the jacket. In this way, the length of the wrought-iron pipe remains

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

constant. To the upper end of this pipe a steel plate is attached, on the lower side of which the knife-edge of the extensometer bears, and which also carries the scale.

The extensometer consists of a long light lever working in a small bearing, at the top of the copper tube, the short end carrying a thin steel knife-edge, and the long end extending to the scale, by means of which the extension of the copper pipe could be measured.

The extension was magnified by this means, so that a movement of 1 millim. of the pointer represented a change in the temperature of the pipe of 0·5°, with the result that the temperature of the pipe could be estimated to $\frac{1}{10}$th of a degree. Before and after any experiment, the temperature corresponding to the given scale reading was determined by actual trial. All the pipes and the exposed parts of the apparatus were carefully lagged with cotton wool and sheet cork to prevent loss or reception of heat.

Measurement of the Temperature of the Water.

For this purpose carefully calibrated thermometers were used, of such a scale that readings within $\frac{1}{100}$th of a degree could be estimated. These were fixed in small brass chambers through which the water passed on entering and leaving the pipes, the chambers being carefully lagged and placed as near the pipe as possible. The chambers were fitted with small stuffing boxes and screw glands to prevent leakage, and are shown in section in fig. 5.

As the difference in pressure between the water entering the pipe and leaving it was small, no correction was found necessary for the observed readings.

In the experiments, at a pressure of about two atmospheres, it was found that the pressure had a small effect on the thermometer readings, due to compression of the bulb, but this was not more than 0·15° C.

To measure the pressure, a "Tee" joint was connected to the top of the copper pipe, as shown in fig. 4. This "tee" carried the extensometer lever in the centre, one branch being connected to the water supply, and the other branch being connected to a mercury pressure gauge in the form of a U-tube.

To regulate the pressure to any desired amount, brass cocks were attached to the waste pipes leading from the pipe and jacket, which could be adjusted to produce the pressure.

The joint between the copper tube and the jacket pipe at the upper end was made by an india-rubber washer and screwed cap, so as to allow free expansion of the pipe. At the lower ends, where the pipe was fixed to the standard, the jacket pipe and inner pipe were soldered together.

Method of Making an Experiment.

In the first place, the inside of the pipe was cleaned by a small brush, then the valves were set so that the required amount of water was passing through the pipe and the jackets. The water in each coil being raised to the same temperature, T_0 say, the scale reading was taken. The initial temperature of the water flowing through the pipe was then set to a given value, say t_1, the temperature of the jacket water being regulated so that the scale reading remained the same as before. When the final temperature of the water flowing through the pipe became steady, its value t_2 was taken. The temperatures of the pipe and jacket water were then again brought to the common value they had initially, and the scale reading again taken. If this agreed with the first reading, the experiment was taken as correct; if not, it was rejected and another made.

The observations would be as follows :—

Experiment I.—Water, 148 grms. per second.

(a.) *Measurement of Surface Temperature* :—

Temperature of jacket water = 47·8°
,, pipe ,, = 47·8° } Scale reading, 6·57 centims.

(b.) *Measurement of Heat transmitted* :—

Initial temperature of water in pipe = 17·98°.
,, ,, ,, jacket = 67·60°.
Final ,, ,, pipe = 24·45°.
,, ,, ,, jacket = 61·10°.

(c.) *Checking Surface Temperature* :—

Temperature of jacket water = 47·9°
,, pipe ,, = 47·7° } Scale reading, 6·56 centims.

Results of the Experiments.

Let

T_0 = temperature of the inner surface of the pipe in degrees Centigrade.
t = mean temperature of the water in the pipe at any cross section.
v = velocity of the water through the pipe in centims. per second.
p = pressure of the water.
r = radius of the pipe in centims.
L = length of the pipe in centims.

Then, for a small element of the internal surface of the pipe $= 2\pi r\,dl$, we may write, for the case of transmission of heat from metal to water,

$$\text{Heat transmitted} = d\text{H},$$
$$= \text{K} \,.\, 2\pi r dl\; \phi\,(\text{T}_0,\, t,\, (\text{T}_0 - t)\,.\, p \,.\, v \,.\, r),$$

where K and the function ϕ are to be determined by experiment.

From the description of the apparatus given above, it will be seen that the effect of each of the quantities T_0, t, $(T_0 - t)$ p, v and r, on the rate of transmission, can be studied separately.

Thus, in determining the effect of the velocity of the water, a series of experiments were made at varying velocities, but in which the values of T_0, t_1, $(T_0 - t_1)$, p and r remained constant.

I.—EFFECT of the Varying Pressures of the Water, T_0, $(T_0 - t_1)$, v, and r constant.

The following table gives the results obtained :—

Diameter of pipes.	Temperature of surface.	Initial temperature of water.	Final temperature of water.	Velocity of water.	Pressure in centims. of mercury.	Rise of temperature.
1·07	60°·4	25°·6	34°·6	116·0	94·0	9°·0
1·07	60·4	25·4	34·3	116·0	113·0	8°·9
1·07	60·4	25·3	34·4	116·0	152·0	9°·1
1·07	60·4	25·3	34·5	116·0	191·0	9°·2

These results show that for the given range of pressure, *i.e.*, from one to two atmospheres, the transmission is practically independent of the pressure.

II.—EFFECT of Variation of Velocity, T_0, $(T_0 - t_1)$, t_1, and r constant.

About 50 experiments of this kind were made. As an example of the results the following table may be given :—

Diameter of pipes.	T_0.	t_1.	t_2.	v.	Rise of temperature.
1·39	47·55	17·98	24·45	69·0	6°·47
1·39	47·55	17·99	24·08	98·0	6°·09
1·39	47·55	18·00	23·92	123·2	5°·92

These results show that the increase in temperature of the water for the given range of temperature, &c., is nearly independent of the velocity. Thus, for an increase in the velocity of 80 per cent., the rise of temperature is only 8·5 per cent. less, or in other words, the heat transmitted for the given range of temperature is nearly proportional to the velocity of the water. Those experiments were repeated, and then tried with a different value of the range $(T_0 - t_1)$, but with the same results, the ranges of temperature varying from 5° to 40°.

III.—EFFECT of Varying Ranges of Temperature t_1, v and r remaining constant.

As an example, the following set of results may be quoted :—

Diameter of pipes.	T_0.	t_1.	t_2.	v.	Rise of temperature.
1·39	47·55	18·00	23·92	186·0	5°·92
1·39	39·35	18·01	22·22	186·0	4°·21
1·39	32·45	18·05	20·83	186·0	2°·78

Now if the heat transmitted under these conditions varied simply in the range of temperature, we should have had for the whole surface of the pipe

$$C\,(T_0 - t)\,ds = W dt,$$

or,

$$C_1 S = W \log \frac{T_0 - t_1}{T_0 - t_2},$$

where

S = whole surface of the pipe,

C_1 = a constant,

W = weight of water flowing through the pipe per second.

Now the values of $\log \frac{T_0 - t_1}{T_0 - t_2}$ for the experiments quoted are ·222, ·218, ·213 respectively, which show that the heat transmitted is proportional to $(T_0 - t)$ multiplied by a function of the temperature T_0.

IV.—EFFECT of Varying the Initial Temperature t_1, the range $(T_0 - t_1)$ v and r being constant.

A set of experiments at 69·0 centims. per second gave :—

Diameter of pipes.	T_0.	t_1.	t_2.	v.	Rise of temperature.
1·39	35·00	18·80	22·18	69·0	3°·38
1·39	53·60	37·40	41·25	69·0	3°·85

These results show a considerable increase in the heat transmitted for the same range and velocity; this increase, as will be seen by comparing Experiments III. and

IV., being greater than would be accounted for by the difference in the value of T_0. Other experiments carefully made gave similar results.

Thus, for a pipe of given diameter these experiments indicate that the transmission of heat from the surface of the pipe to water flowing through it, at velocities above the critical value for the pipe used, is given by an expression of the form

$$dH = K 2\pi r \, dl \, (T_0 - t) \, f(v) \, F(T_0) \, \Phi(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

It is also seen that the values of $F(T_0)$ and $\Phi(t)$ do not vary very much from unity, and may probably be put in the form

$$F(T_0) = 1 + \alpha T_0,$$
$$\Phi(t) = 1 + \beta t,$$

where α and β are constants to be determined by experiment.

Again, from Experiments II., it is seen that the heat transmitted is nearly proportional to the velocity, thus indicating the probable form of the velocity function, as

$$f(v) = V^n,$$

where n is a number a little less than unity, and to be determined by experiments at varying velocity.

It may be noticed that in equation (1) the value of K may depend on the diameter of the pipe and the nature of the surface.

The experiments made on the three drawn copper pipes, of diameter 1·39, 1·07, and ·736 centim., did not clearly indicate what the relation between H and r was, beyond showing that the effect of the variation in diameter of these pipes was not great, the supply of hot water from the heating coils not being sufficient to enable experiments to be made on pipes of larger diameter.

It is shown in the theory that the heat transmitted is proportional to the value of

$$r^{n-2}$$

where n has the value 1·84 approximately.

This would make the heat transmitted across unit area of the surface of the smallest pipe (·736 centim. diameter) about 10 per cent. greater than that transmitted through unit area of the surface of the largest pipe (1·39 centim. diameter) under the same conditions of flow and temperature.

General Theory.

The experiments described in this paper were originally made in order to determine, if possible, an expression for the rate of transmission of heat from metal surfaces to water, without reference to the theory, and which expression has been shown to be of the form given in equation (1).

The Author is indebted to Professor OSBORNE REYNOLDS, who kindly offered to look through the paper before publication, for the following theory of the subject.

The outline of this theory, as has been previously stated, was published in 1874.*

The discovery of the law of resistance in parallel channels, made by Professor REYNOLDS, in 1883,† enables this theory to be definitely stated.

According to this theory, the motion of heat from the surface of the pipe follows the same laws as the motion of momentum to the surface, whether by conduction or convection (though not by radiation and absorption, through the material, which unquestionably plays an important part in the so-called conduction of water).

Taking x as the direction of motion,

r = radius of pipe,
t = temperature of the water,
T_0 = temperature of surface of pipe,
D = weight of unit volume of water,
p = pressure of water per unit area,
W = weight of water discharged per second,
w = velocity of the water flowing through the pipe,
$P = (1 + \cdot 0336t + \cdot 000221t^2)^{-1}$,

A, B, and n constants depending on the nature of the surface.

Then, above the *critical* velocity, the loss of pressure is given by the equation

$$\frac{dp}{dx} = \frac{P^{2-n}}{(2r)^{3-n}} w^n \cdot \frac{B^n}{A} . \ddagger$$

Writing this in the form

$$\pi r^2 \frac{dp}{dx} = \pi r^2 \frac{g}{W} \cdot \frac{P^{2-n}}{(2r)^{3-n}} w^{n-1} \frac{B^n}{A} \cdot \left(\frac{W}{g} w\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

then, in (1), $\pi r^2 (dp/dx)$ is the loss of momentum due to diffusion and convection, so that, according to the above theory, substituting

$$W \frac{dt}{dx} \text{ for } \pi r^2 \frac{dp}{dx},$$

and

$$W (T_0 - t) \text{ for } \frac{W}{g} w,$$

* 'Proc. Manchester Lit. and Phil. Society,' 1874, p. 8.

† 'Phil. Trans.,' 1883, p. 976.

‡ 'Phil. Trans.,' 1883, p. 976.

the equation for the passage of heat will be

$$\frac{\mathrm{W}\,dt}{dx} = \pi r^2 g \frac{\mathrm{P}^{2-n}}{(2r)^{3-n}} w^{n-1} \cdot \frac{\mathrm{B}^n}{\mathrm{A}} \cdot (\mathrm{T}_0 - t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

or writing

$$\mathrm{W} = \mathrm{D} w \pi r^2,$$

the slope of temperature along the pipe is given by

$$\frac{dt}{dx} = \frac{\mathrm{B}^n}{\mathrm{A}} \cdot \frac{g}{\mathrm{D}} \cdot \frac{\mathrm{P}^{2-n}}{(2r)^{3-n}} w^{n-2} (\mathrm{T}_0 - t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

This is supposing that the conductivity of the water, as compared with the viscosity, does not enter; but as it probably does, for ultimately it is conductivity by which the heat passes from the walls of the pipe to the water, there will probably be a coefficient

$$f(c/\mathrm{P}),$$

the form of which can be determined by experiment.

Application of Professor REYNOLDS' *Theory to the Experiments.*

Assuming the variation in the value of t to be small, say, not greater than 6° in the whole length of the pipe, then, integrating equation (3),

$$\log \frac{\mathrm{T}_0 - t_1}{\mathrm{T}_0 - t_2} = \frac{\mathrm{B}^n}{\mathrm{A}} \cdot \frac{g}{\mathrm{D}} \cdot \frac{\overline{\mathrm{P}^{2-n}}}{(2r)^{3-n}} w^{n-2} . \mathrm{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

where

t_1 = initial temperature of the water in the pipe,
t_2 = final ,, ,, ,, ,,
L = length of the pipe,
$\overline{\mathrm{P}^{2-n}}$ = mean value of P^{2-n}, for the water in the pipe.

Now, from equation (4) the value of n can be determined by a set of experiments, in which

$$\overline{\mathrm{P}^{2-n}}$$

has the same value in each.

This was done by plotting the logarithmic homologues of

$$\log \frac{\mathrm{T}_0 - t_1}{\mathrm{T}_0 - t_2} \quad \text{and} \quad w,$$

when it was found that the points plotted all lay approximately on a straight line, there being no systematic deviation.

For the three pipes used in the experiments the slopes of the logarithmic homologues were found to be :—

For pipe No. 1, 1·39 centim. diameter; slope $= -0·14$, or $n = 1·86$.
,, No. 2, 1·07 ,, ,, ,, $= -0·175$, or $n = 1·825$.
,, No. 3, ·736 ,, ,, ,, $= -0·170$, or $n = 1·83$.

In pipe No. 1, the velocities of the water had values between 28·7 and 123·2 centims. per second.

In pipe No. 3, the velocities of the water had values between 60 and 393·7 centims. per second.

It will be seen that the values of n given above correspond with the values we should expect to find for smooth copper pipes from the law of the resistances, the value for glass being about 1·73, and for smooth metal rather higher, rising to a value of 2 for rough metal surfaces.

Effect of Viscosity and Conductivity.

If the conductivity of the water at the bounding surface be neglected, then for experiments at constant velocity equation (4) gives the value of

$$\frac{\log \dfrac{T_0 - t_1}{T_0 - t_2}}{P^{2-n}}$$

constant for different values of t.

Referring to the results given in the tables, it is seen that the value of this expression *rises* with the value of the mean temperature (t_m) of the water, which seems to show that the conductivity of the water at the boundary has an effect.

It was also seen, in the experiments quoted above, p. 79, that the heat transmitted depended on the values of T_0 and t, and that this effect would be represented by coefficients of the form

$$(1 + \alpha T_0) \text{ and } (1 + \beta t).$$

From experiments at constant values of w and t_m, the value of α is found to be

$$\alpha = ·004,$$

and that of β is

$$\beta = ·01.$$

The slope of temperature of the water in the pipe is then given by

$$\frac{dt}{dx} = \frac{B^n}{A} \cdot \frac{g}{D} \cdot \frac{P^{2-n}}{(2r)^{3-n}} w^{n-2} (T_0 - t)(1 + \alpha T_0)(1 + \beta t) \quad . \quad . \quad . \quad (5).$$

Now for smooth metal pipes the value of B^n may be assumed nearly constant, so for a pipe of given length and diameter in which the surface temperature is constant,

$$kL = \frac{(2r)^{3-n}\, w^{2-n} \log \frac{T_0 - t_1}{T_0 - t_2}}{\overline{P^{2-n}}\,(1 + \alpha T_0)(1 + \beta t_m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where k is a constant depending on the nature of the surface of the pipe.

For the ranges of temperature obtained in these experiments no sensible error is introduced by taking the *mean* value of

$$P^{2-n} \text{ and } t$$

for the experiment and substituting them in equation (6).

When the variation in t is considerable, equation (5) must be integrated more exactly.

Applying equation (6) to the results of the experiments on the three copper pipes used, the values of k are found to be :—

Pipe.	Diameter.	Length.	Value of k.			Number of experiments.
			Maximum.	Minimum.	Mean.	
	centims.	centims.				
I.	1·39	47·0	·0108	·0104	·0106	22
II.	1·07	44·5	·0104	·0100	·0102	13
III.	·736	46·0	·0103	·0099	·0100	15

If W = weight of water flowing through the pipe in grammes per second, equation (5) may be written

$$W \frac{dt}{dx} = k\pi r^2 \frac{P^{2-n}}{(2r)^{3-n}} w^{n-1} (T_0 - t)(1 + \alpha T_0)(1 + \beta t) \quad . \quad . \quad . \quad . \quad (7),$$

which gives for the transmission of heat *from metal to water* per square centim. of the surface of the pipe

$$H = \frac{k}{4} \frac{P^{2-n}}{(2r)^{2-n}} (T_0 - t)(1 + \alpha T_0)(1 + \beta t)\, w^{n-1} \quad . \quad . \quad . \quad . \quad . \quad (8),$$

gramme-degrees per second.

Case of Transmission of Heat from Water to Metal.

The theory for this case is the same as in the transmission of heat from the surface to the water, so far as the *convection* of the heat is concerned. It seemed probable that the conductivity coefficients would also be the same, but on experiment this was found not to be so, the viscosity in this case having a much greater effect, the results of the experiments being that the heat transmitted was *nearly* inversely proportional to the mean viscosity of the film of liquid at the surface, so that the conductivity coefficient can be put in the form

$$k/\mathrm{P}_m,$$

where P_m is the mean value of P for the experiment, and

$$\mathrm{P} = \left\{1 + \cdot 0336\left(\frac{\mathrm{T}_0 + t}{2}\right) + \cdot 000221\left(\frac{\mathrm{T}_0 + t}{2}\right)^2\right\}^{-1}.$$

For this case equation (5) now takes the form

$$\frac{dt}{dx} = -\frac{\mathrm{B}^n}{\mathrm{A}} \cdot \frac{g}{\mathrm{D}} \cdot \frac{\mathrm{P}^{2-n}}{(2r)^{3-n}} \cdot \frac{w^{n-2}(t - \mathrm{T}_0)}{\mathrm{P}_m} \quad \quad (9).$$

The results of experiments made under these conditions are given in Table V., and the values of k calculated. These values do not show quite such a high degree of consistency as the values of k deduced from the other experiments, but this is probably due to the difficulty of working under the given conditions. Other experiments in which the heat flowed from water to metal were made, but always with the result that the heat transmitted was sensibly inversely proportional to the mean viscosity of the film at the surface, the deviation being never greater than 7 per cent.

The Transmission of Heat from the Jacket Water to the External Surface of the Pipe.

During all the experiments the temperatures of the jacket water were taken, so that the rate of heat transmission from the jacket water to the surface of the pipe could be calculated.

It was found in this case also that the transmission of heat was sensibly inversely proportional to the mean viscosity of the film of water at the surface of the pipe, the value of the conductivity coefficient being always slightly less than the mean value of P^{-1} for the film of water.

This is shown in Table VI., in which the product in the last term is calculated on the assumption that the effect of velocity and range of temperature is the same as in the previous cases.

It was found from these experiments on the jacket water that the value of n is practically the same as in the case of the pipe, the value for the jacket of No. 1 pipe being 1·855.

The results stated in Tables V. and VI. would, therefore, seem to show that the conductivity coefficients in the passage of heat between a metal surface and water in contact with it, will depend, in their value, on the *direction* of the flow of heat, the viscosity of the film of water at the surface having much less effect when the heat flows from metal to water than when the flow is in the opposite direction.

The experiments were made in the Whitworth Engineering Laboratory, Owens College, in 1895 and 1896, the author being at that time Demonstrator in Engineering in Owens College.

TABLE I.—Copper Pipe, No. I. 47·0 centims. long, 1·39 centim. diameter. Value of $n = 1·86$.

T_0.	t_1.	t_2.	w.	w^{2-n}.	$\log \frac{T_0 - t_1}{T_0 - t_2}$.	$\overline{P^{n-2}}$.	$1+\alpha T_0$.	$1+\beta t_m$.	k.
47·20	18·00	23·92	123·2	1·96	·2263	1·086	1·189	1·209	·0105
39·10	18 01	22·22	123·2	1·96	·2224	1·082	1·156	1·201	·0106
32·28	18·05	20·83	123·2	1·96	·2171	1·079	1·129	1·194	·0105
50·55	33·25	37·00	123·2	1·96	·2440	1·134	1·202	1·351	·0104
40·25	25·10	28·20	123·2	1·96	·2286	1·106	1·161	1·266	·0105
34·50	18·15	21·39	123·2	1·96	·2206	1·080	1·137	1.198	·0107
41·55	18·18	22·85	123·2	1·96	·2227	1·083	1·166	1·205	·0105
47·26	17·99	24·14	98·0	1·90	·2356	1·086	1·189	1·211	·0105
39·15	18·00	22·38	98·0	1·90	·2318	1·082	1·157	1·202	·0107
32·31	18·03	20·90	98·0	1·90	·2241	1·079	1·129	1·195	·0106
45·00	18·79	24·18	98·0	1·90	·2300	1·088	1·180	1·215	·0103
50·25	33·25	37·03	98·0	1·90	·2512	1·134	1·201	1·351	·0104
40·25	25·20	28·40	98·0	1·90	·2387	1·107	1·161	1·268	·0106

TABLE II.—Copper Pipe, No. I. 47·0 centims. long, 1·39 centim. diameter. Value of $n = 1·86$

T_0.	t_1.	t_2.	w.	w^{2-n}.	$\log \frac{T_0 - t_1}{T_0 - t_2}$.	$\overline{P^{n-2}}$.	$1+\alpha T_0$.	$1+\beta t_m$.	k.
47·33	17·98	24·45	69·0	1·80	·2487	1·087	1·189	1·212	·0105
39·20	18·00	22·60	69·0	1·80	·2443	1·083	1·157	1·203	·0107
32·35	18·03	21·03	69·0	1·80	·2348	1·080	1·129	1·195	·0105
45·00	18·77	24·28	83·6	1·86	·2358	1·088	1·180	1·215	·0104
39·65	18·82	23·15	83·6	1·86	·2325	1·086	1·158	1·210	·0104
54·25	42·20	45·10	83·6	1·86	·2750	1·160	1·217	1·436	·0106
39·52	18·32	23·02	58·1	1·767	·2504	1·085	1·158	1·206	·0107
39·55	18·35	23·26	43·6	1·696	·2632	1·085	1·158	1·208	·0108
40·30	18·43	23·65	28·7	1·600	·2725	1·086	1·160	1·210	·0105

TABLE III.—Copper Pipe, No. II. 44·5 centims. long, 1·07 centims. diameter.
Value of $n = 1·825$.

T_0.	t_1.	t_2.	w.	w^{2-n}.	$\log \frac{T_0 - t_1}{T_0 - t_2}$.	$\overline{P^{n-2}}$.	$1 + \alpha T_0$.	$1 + \beta t_m$.	k.
63·35	23·28	32·20	116·0	2·30	·2515	1·139	1·253	1·277	·0100
54·60	23·30	30·12	116·0	2·30	·2455	1·134	1·220	1·267	·0101
47·35	23·30	28·37	116·0	2·30	·2365	1·130	1·189	1·258	·0100
50·85	36·60	39·88	116·0	2·30	·2613	1·184	1·203	1·382	·0104
35·70	21·38	24·35	103·8	2·25	·2321	1·117	1·143	1·248	·0101
56·50	21·39	29·30	97·6	2·23	·2549	1·120	1·226	1·253	·0101
63·25	48·80	52·50	91·6	2·21	·2955	1·230	1·253	1·506	·0104
35·70	21·38	24·40	91·6	2·21	·2366	1·117	1·143	1·229	·0101
52·30	37·60	41·10	88·4	2·19	·2718	1·187	1·209	1·393	·0102
56·50	21·39	29·40	85·3	2·178	·2586	1·128	1·226	1·254	·0100
35·70	21·40	24·50	79·3	2·15	·2438	1·118	1·143	1·229	·0102
63·25	48·80	52·60	79·3	2·15	·3050	1·230	1·253	1·507	·0104
56·50	21·39	29·60	73·1	2·12	·2660	1·130	1·226	1·255	·0101

TABLE IV.—Copper Pipe, No. III. 46 centims. long, 0·736 centim. diameter.
Values of $n = 1·83$.

T_0.	t_1.	t_2.	w.	w^{2-n}.	$\log \frac{T_0 - t_1}{T_0 - t_2}$.	$\overline{P^{n-2}}$.	$1 + \alpha T_0$.	$1 + \beta t_m$.	k.
39·30	14·06	20·72	393·7	2·76	·3060	1·090	1·157	1·174	·0102
31·80	14·06	18·58	393·7	2·76	·2935	1·084	1·125	1·163	·0100
26·25	14·06	17·13	393·7	2 76	·2915	1·081	1·105	1·156	·0102
20·45	14·07	15·64	393·7	2·76	·2820	1·077	1·082	1·149	·0101
17·10	14·07	14·82	393·7	2·76	·2840	1·075	1·068	1·144	·0103
39·30	13·90	20·78	296·0	2·63	·3160	1·089	1·157	1·173	·0100
51·65	37·50	41·80	296·0	2·63	·3620	1·184	1·207	1·397	·0100
42·55	28·33	32·40	296·0	2·63	·3370	1·146	1·170	1·303	·0099
28·20	14·06	17·80	296·0	2·63	·3070	1·082	1·113	1·159	·0100
39·42	13·90	21·00	244·6	2·55	·3255	1·090	1·158	1·174	·0100
26·25	14·04	17·30	244·6	2·55	·3100	1·080	1·105	1·156	·0100
20·55	14·05	15·75	244·6	2·55	·3035	1·077	1·082	1·148	·0100
38·00	14·20	21·80	90·0	2·15	·3840	1·092	1·152	1·180	·0099
56·50	14·20	28·55	90·0	2·15	·4140	1·107	1·227	1·214	·0099
38·00	14·20	22·20	60·0	2·005	·4095	1·093	1·152	1·182	·0099

TABLE V.—Transmission of Heat from Water to Metal. Copper Pipe No. II. 44·5 centims. long, 1·07 centims. diameter. Value of $n = 1·825$.

T_0.	t_1.	t_2.	w.	w^{2-n}.	$\overline{P_m^{-1}}$.	$\log \frac{t_1 - T_0}{t_2 - T}$.	$\overline{P^{n-2}}$.	k.
30·85	52·25	46·60	88·4	2·19	2·7	·3065	1·226	·00743
36·00	52·20	47·80	88·4	2·19	2·85	·3165	1·228	·00726
42·15	52·20	49·20	88·4	2·19	3·03	·3540	1·231	·00778
44·90	52·30	50·05	88·4	2·19	3·12	·3620	1·232	·00763
31·70	53·60	47·60	88·4	2·19	2·75	·3200	1·231	·00765

TABLE VI.—Experiments on Jacket Water of No. I. Pipe. Area of Jacket 0·9 sq. centim. Value of $n = 1·855$.

T_0.	t_1.	t_2.	w.	w^{2-n}.	$\log \frac{t_1 - T_0}{t_2 - T_0}$.	$\overline{P^{n-2}}$.	$\overline{P^{-1}}$.	Values of $\frac{\overline{P} \log \frac{t_1 - T_0}{t_2 - T_0} w^{2-n}}{P^{2-n}}$.
47·90	66·67	60·75	207·0	2·17	·3780	1·225	3·56	·282
39·50	54·21	50·00	207·0	2·17	·3370	1·192	3·00	·291
32·62	43·18	40·40	207·0	2·17	·3050	1·160	2·55	·301
47·85	66·83	60·75	164·0	2·09	·3857	1·225	3·56	·277
39·55	54·13	49·75	164·0	2·09	·3560	1·191	3·00	·295
32·58	42·87	40·00	164·0	2·09	·3270	1·160	2·55	·311
47·77	67·32	60·85	116·0	1·99	·4014	1·226	3·56	·275
39·60	53·95	49·35	116·0	1·99	·3860	1·192	2·99	·306
32·55	43·00	40·00	116·0	1·99	·3380	1·16	2·55	·305
48·00	67·87	60·75	73·0	1·86	·4425	1·227	3·57	·282
39·75	54·91	50·00	73·0	1·86	·3910	1·192	3·01	·288